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Sr. No. of Question Paper : 2594 GC-3 Your Roll No.....

Unique Paper Code : 32355301

Name of the Paper : Differential Equations

Name of the Course : Generic Elective – 3 for Hons. Courses, Under CBCS

Semester : III

Duration : 3 Hours

Maximum Marks : 75

Instructions for Candidates

1. Write your Roll No. on the top immediately on the receipt of this question paper.
2. Attempt all questions by selecting any two parts from each question.

1. (a) Find an integrating factor and solve the differential equation :

$$(e^{x+y} - y)dx + (xe^{x+y} + 1)dy = 0. \quad (6.5)$$

- (b) Solve the equation : $y' + (x + 1)y = e^{x^2} y^3$, $y(0) = 0.5$. (6.5)

- (c) Find the orthogonal trajectories of the family of parabolas $y = ce^{-3x}$. (6.5)

2. (a) Show that e^{3x} and xe^{3x} form a basis of the following differential equation $y'' - 6y' + 9y = 0$. Find also the solution that satisfies the conditions $y(0) = -1.4$, $y'(0) = 4.6$. (6)

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- (b) Solve the initial value problem : (6)

$$x^2y'' + 3xy' + y = 0, y(1) = 4, y'(1) = -2.$$

- (c) Find the radius of convergence of the series : $\sum_{m=2}^{\infty} \frac{(-1)^m (x-1)^{2m}}{4^m}$ (6)

3. (a) Find a general solution of the following nonhomogeneous differential equation :

$$y'' + 3y' + 2y = 30e^{2x}$$

using variation of parameters. (6.5)

- (b) Use the method of undetermined coefficients to find the particular solution of the differential equation : $y'' - 4y' + 4y = 2e^{2x}$. (6.5)

- (c) Find a homogeneous linear ordinary differential equation for which two functions x^3 and x^{-2} are solutions. Show also linear independence by considering their Wronskian. (6.5)

4. (a) Find the general solution of the partial differential equation

$$yzu_x - xzu_y + xy(x^2 + y^2)u_z = 0. \quad (6)$$

- (b) Find a general solution of the differential equation :

$$(x^2D^2 + xD - 4I)y = 0, \text{ where } D = \frac{d}{dx}. \quad (6)$$

- (c) Find the particular solution of the linear system that satisfies the stated initial conditions :

$$\frac{dy_1}{dt} = -5y_1 + 2y_2, \quad y_1(0) = 1$$

$$\frac{dy_2}{dt} = 2y_1 - 2y_2, \quad y_2(0) = -2 \quad \dots (6)$$

5. (a) Find a power series solution of the following differential equation, in powers of x

$$y'' + xy' - 2y = 0. \quad (6.5)$$

- (b) Find the solution of the quasi-linear partial differential equation :

$$u(x+y)u_x + u(x-y)u_y = x^2 + y^2$$

with the Cauchy data $u = 0$ on $y = 2x$. (6.5)

- (c) Reduce the equation : $yu_x + u_y = x$

to canonical form, and obtain the general solution. (6.5)

6. (a) Solve the initial-value problem :

$$u_x + 2u_y = 0, \quad \mu(0, y) = 4e^{-2y}$$

using the method of separation of variables. (6)

- (b) Obtain the canonical form of the equation : $x^2 u_{xx} + 2xyu_{xy} + y^2 u_{yy} = 0$, and hence find the general solution. (6)

(c) Reduce the following partial differential equation with constant coefficients,

$$u_{xx} + 2u_{xy} + 5u_{yy} + u_x = 0.$$

into canonical form.

(6)